ORIGINAL PAPER

STATISTICAL PREDICTION USING TO FORECAST THE THEORY OF MARKOV CHAINS

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Abstract.*The number of students enrolled in the technological studies has seen a downward trend in recent years, with significant decreases in the field of natural resource and environmental protection specialization. This paper uses the theory of Markov chains to forecast the number of students enrolled in pre-university education, upper secoundary, by groups of studies and profile. Knowing future trends allow an appropriate educational strategy based on reality and the requirements of the labor market.*

Keywords: *educational strategy, Markov chains theory, probability.*

1. INTRODUCTION

The main objective of this paper is to determinate the probable number of students enrolled in pre-university education by groups of studies and profile, which will make a major contribution to achieving the educational offer of school organizations. A dynamic process that proposes changes to permanently adapt the organization to changes in the environment in which it evolves [1], the process of strategic management is a set of decisions and managerial actions defining the direction and long-term outcomes of the school organization [2]. School Strategic Management is the set of decisions and actions used to formulate and implement strategies and action plans and is understood as a systematic, interactive process to create the best conditions for the organization, proposing the following steps:analyzing the external environment of the school organization; analysis and assessment of the internal environment; adapting the school development strategy and drawing directions of evolution; defining and selecting variables subject to planning; implementing programs proposed by the organization, performance evaluation [3]. Strategic management presents at the behavioral level the advantage that strategic decisions taken in the group reflect more easily the favorable alternatives, the motivation of the subordinates can also be improved, and the resistance to change can be done by asking for the help of the participants in the elaboration of the strategy. Achieving certain goals is conditional on co-ordinating more actions related to the human resource. Any organization may start from the assumption that the result of a common work activity on the part of the employees is superior to the aggregate individual activities [4].

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According to Lukacs E. [5], both the level of knowledge and the training of the labor force is an important factor in the performance of the organization, and the improvement of a capital investment in maximizing the potential of human resources. Changing circumstances and ongoing management efforts to improve the work challenge organizations to continually evolve - a condition that makes the task of achieving a strategy a work in progress rather than a momentary event [6].

2. MATERIALS AND METHODS

In order to accomplish the present research, we used as a research method the theory of Markov chains [7] to forecast the number of students enrolled in pre-university education by groups of studies and profile. We analyzed the data from the statistical yearbook of Romania between 2015 and 2017 (2017 being the last year updated).

2.1. MATERIALS

The data used is shown in Table 1.

Table 1. The dynamics of students enrolled in upper secoundary education.

Students enrolled (Upper secoundary education, by type of studies and profiles)	**Years (Number of persons)**			
	2013/ 2014	**2014/ 2015**	**2015/ 2016**	**2016/ 2017**
Sciences	206517	200401	189686	187216
Humanities	136700	139346	134953	134441
Special education	392	393	329	336
Theoretical studies - total	**343609**	**340140**	**324968**	**321993**
Technical profile	191675	158208	135516	123086
Natural resources and environment protection profile	59262	54000	49207	45679
Services	128077	120874	112078	107867
Special education	4542	3571	2689	1509
Technological studies - total	**383556**	**336653**	**299490**	**278141**
Pedagogy	7783	8094	8234	8994
Arts education	14336	14352	13962	14183
Sports	15989	16806	16527	16815
Military	1332	1337	1351	1554
Theology	10011	9690	9083	9152
Vocational education-total	**49451**	**50279**	**49157**	**50698**
Students enrolled - total	**776616**	**727072**	**673615**	**650832**

Irrespective of the educational profile, the dynamics of the number of students enrolled in upper secondary education registered a considerable decrease during the analyzed period; the highest decreases are recorded for technological studies, from 383556 total students in 2013-2014 to 278141 in 2016-2017. All the profiles from the vocational education registered a slight increase, from 49451 total students in 2013-2014 to 50698 in 2016-2017.

2.2. METHODS

Stochastic processes represent an important branch of probability theory, with applications in both mathematics and physics, economics, finance, biology, medicine, engineering, etc. An important category of stochastic processes is the Markov processes [8]. Conceptually, a Markov process is the probabilistic analog of processes in classical mechanics, where future development is completely determined by the present state and it is independent of how developed the present state is. It is called a Markov chain of random variables, the string of random variables $(f_n)_{n\in N}$ satisfying the conditions: $(\forall)\ 0\leq t_1\leq\ldots\leq t_n$, $n\geq 2$ and $(\forall)\ i_1,\ldots, i_n \in I$, with I = set of process conditions, we have:

$$P\ (f_{t_n}\ (\zeta) = i_n \mid f_{t_{n-1}}\ (\zeta) = i_{n-1}\ ,\ldots\ldots,\ f_{t_1}\ (\zeta) = i_1\ ) = P\ (f_{t_n}\ (\zeta) = i_n \mid f_{t_{n-1}}\ (\zeta) = i_{n-1}\ ) \quad (1)$$

The equality (1) it is called Markov's property and it is equivalent to the equality:

$$P\ (f_n\ (\zeta) = i_n \mid f_{n-1}\ (\zeta) = i_{n-1}\ ,\ldots\ldots,\ f_1\ (\zeta) = i_1\ ) = P\ (f_n\ (\zeta) = i_n \mid f_{n-1}\ (\zeta) = i_{n-1}\ ),$$

$$(\forall)\ n\ \in N^* \quad (2)$$

The probabilities $P\ (f_t\ (\zeta) = i_t \mid f_{t-1}\ (\zeta) = i_{t-1}\ )$ are called transition probabilities for Markov chain of random variables, they are denoted byp $(t;\ i_{t-1}, i_t\ ), t = \overline{1,n}$.

The significance for $p\ (t;\ i_{t-1}, i_t\ )$ is that of the probability of transition from the state i_{t-1} at the t-1 moment to the state i_t at the t moment.

Markov chain of random variables is uniform if:

$$p\ (t;\ i_{t-1}, i_t\ ) = p_{i_{t-1}\ i_t} \quad (3)$$

The possibility of the occurrence of the $i_t$state at the t moment subject to the occurrence of i_{t-1} state at moment t-1 does not depend on t explicitly.

$$P\ (f_1\ (\zeta) = j \mid f_{t-1}\ = i) = p_{ij} \quad (4)$$

Do not depend on the moments of time corresponding to states, but on the distance in time between states.It results that:

$$\sum_{j\in I} p_{ij} = 1, p_{ij}\ \geq 0,\ i,j\ \in I \quad (5)$$

The matrix whose elements are the probabilities of transition it is called transition matrix and it is denoted by $\prod = (p_{ij})_{i,j\in I}$.

3. RESULTS AND DISCUSSION

Using the theory of Markov chains, we aim to realize a prediction of structure of the number of students enrolled in higher upper secoundary education for the years 2018 and 2019 by type of studies and profiles.

3.1. RESULTS

In the Table 2 below the authors calculated the share of students enrolled in upper secoundary education by type of studies and profiles.

The dynamics of the number of students enrolled in upper secondary education registered a considerable decrease during the analyzed period; instead all the profiles from the vocational education registered a slight increase.

For each pair of consecutive periods of time (t-1/t) = (2014/2015, 2015/2016, 2016/2017), is calculated the partial matrices of the transition.

Table 2. The share of students enrolled in upper secoundary education.

Students enrolled (Upper secoundary education, by type of studies and profiles)	Years (%)			
	2013/2014	**2014/2015**	**2015/2016**	**2016/2017**
Sciences	26.59	27.56	28.16	28.77
Humanities	17.60	19.17	20.03	20.66
Special education	0.05	0.05	0.05	0.05
Theoretical studies – total	**44.24**	**46.78**	**48.24**	**49.47**
Technical profile	24.68	21.76	20.12	18.91
Natural resources and environment protection profile	7.63	7.43	7.30	7.02
Services	16.49	16.62	16.64	16.57
Special education	0.58	0.49	0.40	0.23
Technological studies – total	**49.39**	**46.30**	**44.46**	**42.74**
Pedagogy	1.00	1.11	1.22	1.38
Arts education	1.85	1.97	2.07	2.18
Sports	2.06	2.31	2.45	2.58
Military	0.17	0.18	0.20	0.24
Theology	1.29	1.33	1.35	1.41
Vocational education	**6.37**	**6.92**	**7.30**	**7.79**
Students enrolled – total	**100**	**100**	**100**	**100**

These are square matrices (12x12) denoted by: $G^{t-1/t} = (g_{ij}^{t-1/t})_{ij=\overline{1,12}}$; we denote A $=(a_{ij})_{i=\overline{1,4};j=\overline{1,12}}$; the matrix whose elements are values in (Table 2).

The matrix elements $G^{2013-2014/2014-2015} = (g_{ij}^{2013-2014/2014-2015})_{ij=\overline{1,12}}$ is determined as follows:

- for i = j : $(g_{ij}^{2013-2014/2014-2015})_{ij=\overline{1,12}} = \min(a_{i1}^{2013-2014}, a_{i2}^{2014-2015})$

The differences $(a_{i1}^{2013-2014} - g_{ii}^{2013-2014/2014-2015})$, $i=\overline{1,12}$ are called negative deviations (ND) and the differences $(a_{i2}^{2013-2014} - g_{ii}^{2013-2014/2014-2015})$, $i=\overline{1,12}$ are called positive deviations (PD).

- for i≠ j: $(g_{ij}^{2013-2014/2014-2015})_{ij=\overline{1,12}} = (a_{i1}^{2013-2014} - g_{ii}^{2013-2014/2014-2015})$ x $(a_{i2}^{2013-2014} - g_{jj}^{2013-2014/2014-2015}) / \sum positive\ deviations.$

Table 3. Partial matrix of transition from 2013-2014 to 2014-2015.

	1	2	3	4	5	6	7	8	9	10	11	12	ND
1	26.5919	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
2	0.0000	17.6020	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
3	0.0000	0.0000	0.0505	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
4	0.8811	1.4189	0.0032	21.7596	0.0000	0.1208	0.0000	0.1008	0.1162	0.2293	0.0112	0.0397	2.9212
5	0.0615	0.0990	0.0002	0.0000	7.4270	0.0084	0.0000	0.0070	0.0081	0.0160	0.0008	0.0028	0.2037
6	0.0000	0.0000	0.0000	0.0000	0.0000	16.4917	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
7	0.0283	0.0455	0.0001	0.0000	0.0000	0.0039	0.4911	0.0032	0.0037	0.0074	0.0004	0.0013	0.0937
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0022	0.0000	0.0000	0.0000	0.0000	
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.8460	0.0000	0.0000	0.0000	
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	2.0588	0.0000	0.0000	
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1715	0.0000	
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.2891	
PD	0.9708	1.5634	0.0036			0.1331		0.1111	0.1280	0.2527	0.0124	0.0437	

The matrix whose elements are the values in the table above represent $G^{2013\text{-}2014/2014\text{-}2015}$. As observed in Table 3, the type of studies and profiles who lost percentages are: Technical profile (-2.9212), Natural resources and environment protection profile (-0.2037). All theoretical and vocational education profiles won percentage: Sciences (+0.9708), Humanities (+1.5634), Pedagogy (+0.1111), Arts education (+0.1280), Sports (+0.2527), Military (+0.0124), Theology (+0.0437). Also with a positive deviation are services profiles (+0.1331). Proceeding analog, the following transition matrices are obtained for $G^{2014\text{-}2015/2015\text{-}2016}$ (Table 4).

Table 4. Partial matrix of transition from 2014-2015 to 2015-2016

	1	2	3	4	5	6	7	8	9	10	11	12	ND
1	27.5627	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
2	0.0000	19.1654	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
3	0.0017	0.0024	0.0488	0.0000	0.0000	0.0000	0.0000	0.0003	0.0003	0.0004	0.0000	0.0000	0.0052
4	0.5264	0.7664	0.0000	20.1177	0.0000	0.0119	0.0000	0.0963	0.0871	0.1253	0.0147	0.0138	1.6419
5	0.0392	0.0570	0.0000	0.0000	7.3049	0.0009	0.0000	0.0072	0.0065	0.0093	0.0011	0.0010	0.1221
6	0.0000	0.0000	0.0000	0.0000	0.0000	16.6248	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
7	0.0295	0.0429	0.0000	0.0000	0.0000	0.0007	0.3992	0.0054	0.0049	0.0070	0.0008	0.0008	0.0920
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.1132	0.0000	0.0000	0.0000	0.0000	
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.9739	0.0000	0.0000	0.0000	
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	2.3115	0.0000	0.0000	
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1839	0.0000	
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.3327	
PD	0.5967	0.8688				0.0135		0.1091	0.0988	0.1420	0.0167	0.0157	

As observed in Table 4, the type of studies and profiles who lost percentages is: Technical profile (-2.9212), Natural resources and environment protection profile (-0.2037), Special education (-0.0920). As in the previous partial matrix of transition, theoretical and vocational education profiles won percentage: Sciences (+0.5967), Humanities (+0.8688), Pedagogy (+0.1091), Arts education (+0.0988), Sports (+0.1420), Military (+0.0167), Theology (+0.0157). Though with low values, the services profile continues to gain

percentages (+0.0135). Proceeding analog, the following transition matrices are obtained for $G^{2015\text{-}2016/2016\text{-}2017}$(Table 5).

Table 5. Partial matrix of transition from 2015-2016 to 2016-2017.

	1	2	3	4	5	6	7	8	9	10	11	12	ND
1	28.1594	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
2	0.0000	20.0341	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
3	0.0000	0.0000	0.0488	0.0000	0,0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
4	0.4240	0.4355	0.0019	18.9121	0.0000	0.0000	0.0000	0.1116	0.0745	0.0910	0.0267	0.0404	1.2056
5	0.1007	0.1034	0.0005	0.0000	7.0186	0.0000	0.0000	0.0265	0.0177	0.0216	0.0063	0.0096	0.2864
6	0.0227	0.0233	0.0001	0.0000	0.0000	16.5737	0.0000	0.0060	0.0040	0.0049	0,0014	0.0022	0.0646
7	0.0588	0.0604	0.0003	0.0000	0.0000	0.0000	0.2319	0.0155	0.0103	0.0126	0.0037	0.0056	0.1673
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.2224	0.0000	0.0000	0.0000	0.0000	
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	2.0727	0.0000	0.0000	0.0000	
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	2.4535	0.0000	0.0000	
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2006	0.0000	
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.3484	
PD	0.6062	0.6226	0.0028					0.1596	0.1065	0.1301	0.0382	0.0578	

The service profile registered for the first time negative deviations (-0.0646), the technical profile and natural resources and environment protection profilecontinue to lose percentage and all theoretical and vocational education profiles won percentage again. It is calculated the total matrix of transition for 2013-2017 by summing the three partial matrices obtained previously. The total matrix of transition for the period 2013-2017, $G^{2013\text{-}2017}$ has as elements the values in (Table 6).

Table 6. Total matrix of transition for the period 2013-2017

	1	2	3	4	5	6	7	8	9	10	11	12	Total
1	82.3141	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	82.3141
2	0.0000	56.8015	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	56.8015
3	0.0017	0.0024	0.1482	0.0000	0.0000	0.0000	0.0000	0.0003	0.0003	0.0004	0.0000	0.0000	0.1534
4	1.8315	2.6207	0.0052	60.7894	0.0000	0.1327	0.0000	0.3087	0.2778	0.4456	0.0527	0.0939	66.5581
5	0.2013	0.2594	0.0007	0.0000	21.7505	0.0093	0.0000	0.0407	0.0323	0.0469	0.0082	0.0134	22.3628
6	0.0227	0.0233	0.0001	0.0000	0.0000	49.6901	0.0000	0.0060	0.0040	0.0049	0.0014	0.0022	49.7547
7	0.1166	0.1489	0.0004	0.0000	0.0000	0.0045	1.1222	0.0241	0.0189	0.0270	0.0049	0.0077	1.4752
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	3.3378	0.0000	0.0000	0.0000	0.0000	3.3378
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	5.8926	0.0000	0.0000	0.0000	5.8926
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	6.8237	0.0000	0.0000	6.8237
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5560	0.0000	0.5560
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	3.9702	3.9702
Total	84.4878	59.8563	0.1545	60.7894	21.7505	49.8368	1.1222	3.7175	6.2259	7.3486	0.6232	4.0873	

3.2. DISCUSSION

To predict structure for the year 2018,it is calculated the probability of transition matrix, based on matrix $G^{2013\text{-}2017}$, by dividing each element of the matrix $G^{2013\text{-}2017}$ at the sum of the line on which that item is.We obtain the matrix denoted by $GP^{2013\text{-}2017}=(g_{ij}^{2013/2017})_{ij=\overline{1,12}}$, whose elements are the values in the table below (Table 7).

Table 7. Probability of transition matrix.

	1	2	3	4	5	6	7	8	9	10	11	12
1	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0,0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0,0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0109	0.0159	0.9660	0.0000	0.0000	0.0002	0,0000	0.0020	0.0018	0.0026	0.0003	0.0003
4	0.0275	0.0394	0.0001	0.9133	0.0000	0.0020	0,0000	0.0046	0.0042	0.0067	0.0008	0.0014
5	0.0090	0.0116	0.0000	0.0000	0.9726	0.0004	0,0000	0.0018	0.0014	0.0021	0.0004	0.0006
6	0.0005	0.0005	0.0000	0.0000	0.0000	0.9987	0,0000	0.0001	0.0001	0.0001	0.0000	0.0000
7	0.0790	0.1009	0.0003	0.0000	0.0000	0.0031	0,7607	0.0163	0.0128	0.0183	0.0033	0.0052
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0,0000	1.0000	0.0000	0.0000	0.0000	0.0000
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0,0000	0.0000	1.0000	0.0000	0.0000	0.0000
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0,0000	0.0000	0.0000	1.0000	0.0000	0.0000
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0,0000	0.0000	0.0000	0.0000	1.0000	0.0000
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0,0000	0.0000	0.0000	0.0000	0.0000	1.0000

The sum of the elements on each line is equal to 1, rule of probabilities. The structure predicted for the year 2017-2018 it is calculated as product between matrix transpose $GP^{2013\text{-}2017}$ and the vector below, representing the share of students enrolled in upper secoundary education by types of studies and profile in 2016-2017(see last column Table 2):

$$\begin{pmatrix} 28.76564 \\ 20.65679 \\ 0.051626 \\ 18.91210 \\ 7.018555 \\ 16.57371 \\ 0.231857 \\ 1.381923 \\ 2.179211 \\ 2.583616 \\ 0.238771 \\ 1.406200 \end{pmatrix}$$

Therefore, the projected structure of the number of students enrolled in upper secondaryeducation in 2017-2018, by type of studies and profiles is:

Table 8. Projected structure of thenumber of students enrolled in upper secondary education in 2017-2018, by type of studies and profiles (%)

	Students enrolled	**2018**
1	Sciences	29.93410851
2	Humanities	22.30015862
3	Special education in Theoretical studies	0.051625578
4	Technical track	15.77588569
5	Natural resources and environment protection track	6.639510082
6	Services	16.61088034
7	Special education in Technological studies	0.134173007
8	Pedagogy	1.585695957
9	Arts education	2.358227974
10	Sports	2.865834636
11	Military	0.274810831
12	Theology	1.469088772
	Students enrolled - total	100

As observed in table above, the projected structure of the number of students enrolled in upper secondary education in 2018, keeps the growth trend for sciences and humanities profiles, also for all profiles of vocational education and the downward trend for technical and natural resources and environment protection profiles.

To make a prediction for the next year, it was determined the partial matrix of transition for 2018/2019 and the total matrix of transition for the period 2013-2019.Thus, $G^{2018\text{-}2018}$ has as elements the values in (Table 9).

Table 9. Partial matrix of transition from 2017-2018 to 2018-2019.

	1	2	3	4	5	6	7	8	9	10	11	12	ND
1	29.3757	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	21.5149	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	00000	0.0000	0.0000	0.0000	0.0000
3	0.0000	0.0000	0.0516	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.4843	0.6811	0.0000	15.7759	0.0000	0.0150	0.0000	0.0845	0.0742	0.1170	0.0149	0.0261	1.4971
5	0.0605	0.0850	0.0000	0.0000	6.6395	0.0019	0.0000	0.0105	0.0093	0.0146	0.0019	0.0033	0.1869
6	0.0000	0.0000	0.0000	0.0000	0.0000	16.5936	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	0.0137	0.0192	0.0000	0.0000	0.0000	0.0004	0.1342	0.0024	0.0021	0.0033	0.0004	0.0007	0.0422
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.4883	0.0000	0.0000	0.0000	0.0000	0.0000
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	2.2727	0.0000	0.0000	0.0000	0.0000
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	2.7310	0.0000	0.0000	0.0000
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2576	0.0000	0.0000
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.4390	0.0000
PD	0.5584	0.7853	0.0000	0.0000	0.0000	0.0173	0.0000	0.0974	0.0856	0.1349	0.0172	0.0301	1.7262

With the exception of the services profile which increased percentage (+0.0173), all profiles from technological studies record have decreased in percentage (-1.4971 for technical profile, -0.1869 for natural resourcesand environment protection profile, -0.0422 for special education in technological studies). Sciences and humanities profiles and all the profiles from vocational education registered increases in percentage (see PD-positive deviations, from Table 9). The total matrix of transition for the period 2013-2019, $G^{2013\text{-}2019}$ has as elements the values in the table below (Table 10).

Table 10. Total matrix of transition for the period 2013-2019.

	1	2	3	4	5	6	7	8	9	10	11	12	Total
1	140.4554	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	140.4554
2	0.0000	98.9732	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0,0000	98.9732
3	0.0017	0.0024	0.2514	0.0000	0.0000	0.0000	0.0000	0.0003	0.0003	0.0004	0.0000	0.0000	0.2567
4	2.8457	4.0473	0.0052	93.8383	0.0000	0.1650	0.0000	0.4855	0.4332	0.6906	0.0839	0.1485	102.7432
5	0.3239	0.4318	0.0007	0.0000	35.2164	0.0132	0.0000	0.0621	0.0511	0.0765	0.0120	0.0200	36.2077
6	0.0227	0.0233	0.0001	0.0000	0.0000	82.8574	0.0000	0.0060	0.0040	0.0049	0.0014	0.0022	82.9220
7	0.1482	0.1933	0.0004	0.0000	0.0000	0.0055	14327	0.0296	0.0238	0.0346	0.0059	0.0094	1.8834
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	6.2080	0.0000	0.0000	0.0000	0.0000	6.2080
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	10.3445	0.0000	0.0000	0.0000	10.3445
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	12.1383	0.0000	0.0000	12.1383
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0523	0.0000	1.0523
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	6.8154	6.8154
Total	143.7976	103.6713	0.2578	93.8383	35.2164	83.0412	1.4327	6.7915	10.8567	12.9454	1.1556	6.9954	

It is calculated the predicted structure for the year 2019. The matrix of transition probabilities $GP^{2013\text{-}2019}$ has as elements the values in table below (Table 11).

Table 11. Probability of transition matrix.

	1	2	3	4	5	6	7	8	9	10	11	12
1	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0066	0.0095	0.9796	0.0000	0.0000	0.0001	0.0000	0.0012	0.0011	0.0016	0.0002	0.0002
4	0.0277	0.0394	0.0001	0.9133	0.0000	0.0016	0.0000	0.0047	0.0042	0.0067	0.0008	0.0014
5	0.0089	0.0119	0.0000	0.0000	0.9726	0.0004	0.0000	0.0017	0.0014	0.0021	0.0003	0.0006
6	0.0003	0.0003	0.0000	0.0000	0.0000	0.9992	0.0000	0.0001	0.0000	0.0001	0.0000	0.0000
7	0.0787	0.1026	0.0002	0.0000	0.0000	0.0029	0.7607	0.0157	0.0126	0.0184	0.0031	0.0050
8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000
9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000
10	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000
11	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000
12	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

The structure predicted for the year 2019 is calculated as a product between matrix transpose $GP^{2013\text{-}2019}$ and the vector belowrepresenting the share of students enrolled inupper secoundary education by types of studies and profiles in 2018(see last column Table 8):

$$\begin{pmatrix} 29.93411 \\ 22.30016 \\ 0.051626 \\ 15.77589 \\ 6.63951 \\ 16.61088 \\ 0.134173 \\ 1.585696 \\ 2.358228 \\ 2.865835 \\ 0.274811 \\ 1.469089 \end{pmatrix}$$

Therefore, the projected structure of the number of students enrolled in upper secondary education in 2019, by type of studies and profiles is:

Table 12. Projected structure of thenumber of students enrolled in upper secondary education in 2019, by type of studies and profiles(%)

	Students enrolled	**2019**
1	Sciences	30.44589962
2	Humanities	23.01972107
3	Special education in Theoretical studies	0.051547201
4	Technical track	14.40856553
5	Natural resources and environment protection track	6.457734821
6	Services	16.62610218
7	Special education in Technological studies	0.102067519
8	Pedagogy	1.675002275
9	Arts education	2.436650577
10	Sports	2.989430899
11	Military	0.290616179
12	Theology	1.496662128
	Students enrolled - total	100

As observed in table above, the projected structure of the number of students enrolled in upper secondary education in 2019 keeps the slight growth trend for sciences and humanities profile, also for all profiles of vocational education and the downward trend for technical and natural resources and environment protection profiles, the services profile shows a slight increase in the percentage.

4. CONCLUSIONS

The predictions made using Markov chains theory provides us with information regarding the percentage of the number of students enrolled in pre-university education by groups of studies and profile, which will make a major contribution to achieving the educational offer of school organizations. In the last years, the number of students enrolled in technological studies, especially for technical profiles, natural resources and environment protection profile, registered a major drop. The forecasts calculated above show that this descending trend will be maintained. Regarding profiles in vocational education, they registred constant increase in the analyzed period and maintained a growth trend according the forecasts calculated above.

Using Markov chains theory, forecasts can be made in all areas of interest, regardless of the nature or object of activity of the organization. For school organizations, knowing future trends allows the development of an adequate educational strategy based on reality of the demographic structure and the requirements of the labor market.

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